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SHOCK SPECTRA DESIGN METHODS FOR EQUIPMENT IN IMPULSIVELY LOADED STRUCTURES

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An analytical method is developed whereby a simple estimate can be obtained of the maximum dynamic response of light equipment attached to a structure subjected to ground motion. The natural frequency of the equipment, modeled as a single-degree-of-freedom system, is considered to be close or equal to one of the natural frequencies of the N-degree-of-freedom structure. This estimate provides a convenient, rational basis for the structural design of the equipment and its installation.

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20. ABSTRACT (Continued)

The approach is based on the transient analysis of lightly damped tuned or slightly nontuned equipment-structure systems in which the mass of the equipment is much smaller than that of the structure. It is assumed that the information available to the designer is a design spectrum for the ground motion, fixed-base modal properties of the structure, and fixed-base properties of the equipment. The results obtained are simple estimates of the maximum acceleration and displacement of the equipment. The method can also be used to treat closely-space modes in structural systems, where the square root of the sum of the squares procedure is known to be invalid.

This analytical method is also applied to nontuned equipment-structure systems for which the conventional floor spectrum method is mathematically valid. A closed-form solution is obtained which permits an estimate of the maximum response of the equipment to be determined without the necessity to compute time histories as required by the floor spectrum method.

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INTRODUCTION

In this report we present a shock spectrum approach to the design of lightly damped relatively light equipment in structures subjected to ground shock. The analysis uses as a model an N-degree-of-freedom structure to which is attached a single-degree-of-freedom component. It is known that significant interaction effects occur when the equipment frequency is close or equal to one of the natural frequencies of the structure, referred to as tuning. Otherwise, the conventional floor spectrum method which neglects interaction is valid.

In previous research the undamped equipment-structure problem for tuned systems [1] and slightly nontuned systems [2] was studied and it was shown that a design ground response spectrum together with fixed-base dynamic properties of the structure alone and of the equipment alone can be used to estimate the peak response of the equipment. The analysis took advantage of the mathematical structure of the equations and of asymptotic methods made possible by the smallness of the equipment mass in comparison with the mass of the structure to obtain simple results for tuned and nearly tuned systems. In this paper we extend the results to damped tuned, nearly tuned, and completely nontuned systems. For light equipment and small damping, the results obtained can be easily and efficiently implemented by a designer. The most important aspect of the analysis is its extreme simplicity; namely, if the response spectrum for the ground motion is available, the peak response of the equipment can be calculated merely by multiplying the former by an amplification factor.

This approach is in contrast to several earlier analyses of equipment response, such as the floor spectrum method in which the equipment is treated as a single-degree-of-freedom system subjected to a base motion that is taken to be that which the structure would experience at the attachment point in the absence of equipment. Not only does this method neglect interaction, it has the further disadvantage of requiring that an expensive time-history analysis of the structure be conducted in order to determine a base motion [3]. Approximate techniques that bypass associated computational problems have been proposed whereby floor response spectra are developed from ground spectra, but these methods are *ad hoc* and their accuracy cannot be evaluated.

It is also possible to account for interaction by considering the system to be an $N+1$-degree-of-freedom which is then treated by a modal analysis using an ensemble of spectrum-consistent ground motion records. Neither approach is, however, ideal. If equipment-structure interaction is important, two closely spaced modes will appear, the contributions of which must be summed when a modal approach is used, but there exists no consensus as to an appropriate summation procedure. A disadvantage of the second approach--time-history analysis--is that it is very expensive and for a given item of equipment must be carried out for a wide range of ground motion. If the equipment is light, the use of an $N+1$-degree-of-freedom model with standard structural dynamics computer codes may mask significant response. The method developed here allows the design engineer to use the given ground motion design spectrum directly to determine the response of the equipment. The expense of time-history analysis and the difficulties of evaluating response when closely spaced modes are involved are thus avoided.

In this report we describe physical and numerical experiments on the response of tuned and nearly tuned equipment-structure systems. The physical experiments were carried out on the large shaking table at the University of California, Berkeley, and involved a 40,000 lb, three-story steel frame model with oscillators simulating items of equipment. The model was subjected to a wide variety of simulated ground motions and the peak accelerations in the equipment items measured. The experiments confirmed the accuracy of the analysis in predicting the response of light equipment.

In addition to the physical experiments we have performed numerical experiments using as a model a standard ten-story building and a single equipment item. The numerical experiments corroborated the results obtained in the physical experiments and indicated some surprising aspects of conservatism in generally accepted modal summation rules. It is shown in this report that both the SRSS summation method and the NRL summation method can significantly overestimate equipment response in damped tuned systems.

The results of the analysis for the lightly damped light equipment in ground shock loaded structures presented here can be used to design the damping in the structure and in the equipment to minimize the response of the equipment. It is shown that the best choice of damping is when the damping in the structure and the equipment are equal.

The results developed here have been mainly concerned with short duration loading typical of ground shock, but by application of random vibration theory to the tuned equipment-structure system it can be shown that for very long duration ground motion, similar predictions of equipment response can be developed. These results could be useful in predicting the response of light equipment in large ground vehicles traversing randomly rough track such as the MX system.

ANALYSIS

The equations of motion of the N-degree-of-freedom structural system to which is attached a single-degree-of-freedom equipment item (Figure 1) take the form

$$\sum_{j=i}^{N}(M_{ij}\ddot{U}_j + C_{ij}\dot{U}_j + K_{ij}U_j) = \sum_{j=1}^{N}(C_{ij}R_j\dot{u}_g + K_{ij}R_ju_g) + Fe_i \ , \quad i=1,2,\ldots,N \tag{1}$$

where M_{ij}, C_{ij}, and K_{ij} are the mass, damping, and stiffness matrices, respectively, and U_i is the absolute displacement of the i^{th} degree of freedom. The vector R_i is a vector of influence coefficients introduced to couple the ground motion $u_g(t)$ to the structure, and e_i a vector whose components are zero at every degree of freedom except the one to which the equipment is attached, denoted by the index r, where it takes unit value. The term F is the interaction force between the equipment item and the structure. The equation of motion for the equipment displacement u is

$$-m\ddot{u} = F = c(\dot{u} - \dot{U}_r) + k(u - U_r) \tag{2}$$

where m, c, and k are the mass, damping, and stiffness of the equipment, respectively. Using classical modal description of the structure and Laplace transform techniques, the solution for $\bar{u}$ for the multidegree-of-freedom system takes the form

$$\bar{u}\left[(p^2+2\beta\omega p+\omega^2) + p^2\sum_{k=1}^{N}\frac{m\Phi_r^{k^2}(2\beta\omega p+\omega^2)}{M_k(p^2+2B_k\Omega_k p+\Omega_k^2)}\right]$$

$$= \sum_{k=1}^{N}\frac{\Phi_r^k\sum_{i=1}^{N}\Phi_i^k\sum_{l=1}^{N}M_{il}R_l(2B_k\Omega_k p+\Omega_k^2)(2\beta\omega p+\omega^2)}{M_k(p^2+2B_k\Omega_k p+\Omega_k^2)}\bar{u}_g \tag{3}$$

where β is the fraction of critical damping for the equipment, Ω_k is the k^{th} natural frequency of the structure alone, Φ_i^k is the i^{th} component of the k^{th} mode shape of the building alone, M_k is the generalized mass for the k^{th} mode, and B_k is the fraction of critical damping in the k^{th} mode assumed small enough not to couple the modal equations.

The solution of eq. (1) will be obtained by residue theory. The poles of the transfer by which $\bar{u}$ is related to $\bar{u}_g$ are simple poles and determined by the zeroes of the denominator of the expression in eq. (1). Two different situations arise, one when the equipment is tuned or nearly tuned and the other when the natural frequency of the equipment is well away from any of the structural frequencies *i.e.*, is grossly nontuned. In both cases the poles, for light equipment, appear near the natural frequencies of the structure alone and of the equipment alone. For tuned or nearly tuned equipment, two closely spaced poles (tuning poles) appear, located near the equipment frequency and the frequency of the structure to which the equipment is nearly tuned, one below those frequencies and one above them. For an undamped perfectly tuned system, these two poles coalesce into a double pole as the equipment mass approaches zero. This gives rise to a response in the time domain that grows without bound proportional

to time regardless of the time history of the input. Thus, the contribution to the sum of the residues at all poles is dominated by the residues at the tuning poles. The contribution of the summation term to the residues at these two poles is dominated by the term $k = n$ since the denominator of that term is nearly zero. Hence, in the region of the tuning poles, which is around $p = i\omega$, eq. (3) can be approximated by

$$\bar{u}\left[(p^2 + 2\beta\omega p + \omega^2) + p^2\gamma\,\frac{2\beta\omega p+\omega^2}{p^2+2B_n\Omega_n p+\Omega_n^2}\right] = \frac{(2\beta\omega p+\omega^2)(2B_n\Omega_n p+\Omega_n^2)}{p^2+2B_n\Omega_n p+\Omega_n^2}\,C_r^n\,\bar{\ddot{u}}_g \tag{4}$$

where

$$\gamma = \frac{m}{M_n/(\Phi_r^n)^2} \tag{5}$$

and

$$C_r^n = \Phi_r^n \sum_{i=1}^{N}\sum_{j=1}^{N} \Phi_i^n M_{ij} R_j / M_n \tag{6}$$

are the effective mass ratio and participation factors, respectively.

The contribution to the response of the nontuned poles is quite standard. Their locations are close to what they would be for the structure alone, namely

$$p = -B_m\Omega_m \pm i\Omega_m \tag{7}$$

Evaluating the residues at these poles and dropping negligible terms, we obtain to dominant order the contributions from the nontuned structure poles:

$$\int_0^t \ddot{u}_g(\tau) \sum_{\substack{m=1\\ m\neq n}}^{N} \frac{C_r^m}{(1-\Omega_m/\omega)^2}\,\Omega_m e^{-B_m\Omega_m(t-\tau)} \sin\Omega_m(t-\tau)\,d\tau \tag{8}$$

The contribution to the response from the tuning poles is different and must be obtained by a special approach. This has two parts; a dominant portion that will be developed subsequently and that has a later-occurring peak, and a nondominant portion that is of the same order as the terms above. The second portion is easily obtained and takes the form

$$\int_0^t \ddot{u}_g(\tau) \sum_{\substack{m=1\\ m\neq n}}^{N} \frac{C_r^m}{(1-\omega/\Omega_m)^2}\,\omega e^{-B\omega(t-\tau)} \sin\omega(t-\tau)\,d\tau \tag{9}$$

The dominant portion of the contribution from the tuning poles given by eq. (4) may be written in the form

$$\bar{u} = [N(p)/D(p)]\,\bar{\ddot{u}}_g \tag{10}$$

where

$$N(p) = (2\beta\omega p + \omega^2)[2B_n(1+\xi)\omega p + (1+\xi)^2\omega^2]C_r^n \tag{11}$$

and

$$\begin{aligned} D(p) = p^4 &+ \omega p^3[2\beta(1+\gamma) + 2B_n(1+\xi)] \\ &+ \omega^2 p^2[2 + \gamma + 2\xi + \xi^2 + 4\beta B_n(1+\xi)] \\ &+ \omega^3 p[2\beta(1+\xi)^2 + 2B_n(1+\xi)] + \omega^4(1+\xi)^2 \end{aligned} \tag{12}$$

In the above, $\xi = (\Omega_n - \omega)/\omega$ is the nontuning parameter.

The roots of $D(p)$ will be close to $p = \pm i\omega$ since γ, β, B_n, and ξ are taken to be small. To locate the poles of $D(p)$, we replace p in Eq. (12) by

$$p = i\omega(1+\delta) \tag{13}$$

where δ is a small quantity in terms of which eq. (12) becomes

$$\begin{aligned} &\delta^4 + \{4 - i[2\beta(1+\gamma) + 2B_n(1+\xi)]\}\,\delta^3 \\ &\quad + \{4 - \gamma - 2\xi - \xi^2 - 4\beta B_n(1+\xi) - i[6\beta(1+\gamma) + 6B_n(1+\xi)]\}\,\delta^2 \\ &\quad + \{-2\gamma - 4\xi - 2\xi^2 - 8\beta B_n(1+\xi) - i[2\beta(2+3\gamma-2\xi-\xi^2) + 4B_n(1+\xi)]\}\,\delta \\ &\quad + \{-\gamma - 4\beta B_n(1+\xi) - i[2\beta(\gamma-2\xi-\xi^2)]\} = 0 \end{aligned} \tag{14}$$

The solution of Eq. (14), to dominant terms, is

$$\delta = \frac{\xi}{2} \pm \frac{\lambda}{2} + i\left(\frac{\beta+B_n}{2} \pm \frac{\mu}{2}\right) \tag{15}$$

where here and throughout the remainder of the analysis the upper signs are taken together to give one root and the lower the other. The quantities λ and μ are given by

$$\lambda = \frac{1}{\sqrt{2}}\left\{\{[\gamma+\xi^2-(\beta-B_n)^2]^2 + 4\xi^2(\beta-B_n)^2\}^{1/2} + [\gamma+\xi^2-(\beta-B_n)^2]\right\}^{1/2} \tag{16}$$

$$\mu = \frac{1}{\sqrt{2}}\left\{\{[\gamma+\xi^2-(\beta-B_n)^2]^2 + 4\xi^2(\beta-B_n)^2\}^{1/2} - [\gamma+\xi^2-(\beta-B_n)^2]\right\}^{1/2} \tag{17}$$

The formal inversion of (10) is

$$\ddot{u}(t) = \frac{1}{2\pi i}\int_{\Gamma} \frac{N(p)}{D(p)}\,\bar{\ddot{u}}_g(p)e^{pt}dp \tag{18}$$

where Γ is a suitable Bromwich path. If $\bar{\ddot{u}}_g(p)$ is taken to be 1, then the inversion directly yields that portion of Green's function $\ddot{u}_G(t)$ for the solution which comes from the dominant contribution of the tuning poles. The solution for the equipment acceleration for given ground motion $\ddot{u}_g(t)$ from the dominant contributions of the tuning poles takes the form

$$\ddot{u}(t) = \int_0^t \ddot{u}_G(t-\tau)\ddot{u}_g(\tau)d\tau \tag{19}$$

The function $\ddot{u}_G$ will be obtained by residue theory, since there are no branch cuts in the p plane. The inversion of $\bar{\ddot{u}}_G$ for the general case (eq. (15)) is obtained by writing the denominator $D(p)$ in the form

$$D(p) = (p-p_1)(p-\bar{p}_1)(p-p_2)(p-\bar{p}_2)$$

where

$$p_1 = i\omega\left(1+\frac{\xi}{2}+\frac{\lambda}{2}\right) - \omega\left(\frac{\beta+B_n}{2}+\frac{\mu}{2}\right)$$

$$p_2 = i\omega\left(1+\frac{\xi}{2}-\frac{\lambda}{2}\right) - \omega\left(\frac{\beta+B_n}{2}-\frac{\mu}{2}\right)$$

and $\bar{p}_1$ and $\bar{p}_2$ are the complex conjugates of p_1 and p_2. Evaluating the residues at each pole and collecting complex conjugate terms in pairs, we obtain the result, correct to dominant order,

$$\ddot{u}_G(t) = C_r^n \frac{\omega}{\lambda^2+\mu^2} e^{-(\beta+B)\omega t/2} [\lambda \sinh\frac{\mu}{2}\omega t \cos\frac{\lambda}{2}\omega t \sin(1+\frac{\xi}{2})\omega t$$

$$- \lambda \cosh\frac{\mu}{2}\omega t \sin\frac{\lambda}{2}\omega t \cos(1+\frac{\xi}{2})\omega t$$

$$- \mu \sinh\frac{\mu}{2}\omega t \cos\frac{\lambda}{2}\omega t \cos(1+\frac{\xi}{2})\omega t$$

$$- \mu \cosh\frac{\mu}{2}\omega t \sin\frac{\lambda}{2}\omega t \sin(1+\frac{\xi}{2})\omega t] \qquad (20)$$

The dominant contribution of the tuning poles to the acceleration response $\ddot{u}(t)$ to a specified imposed ground motion $\ddot{u}_g(t)$ is obtained by substituting the above equation in Eq. (19). The complete solution for the response of the equipment is then obtained by summing eqs. (8), (9), and (19) and takes the form

$$\ddot{u}(t) = \int_0^t \ddot{u}_g(\tau) \left\{ \sum_{\substack{m=1 \\ m \neq n}}^{N} \frac{C_r^m}{1-(\Omega_m/\omega)^2} \Omega_m e^{-B_m \Omega_m (t-\tau)} \sin\Omega_m (t-\tau) \right.$$

$$\left. + \sum_{\substack{m=1 \\ m \neq n}}^{N} \frac{C_r^m}{1-(\omega/\Omega_m)^2} \omega e^{-\beta\omega(t-\tau)} \sin\omega(t-\tau) + \ddot{u}_G(t-\tau) \right\} d\tau \qquad (21)$$

where $\ddot{u}_G$ is given by eq. (20).

The character of the two parts of the solution in Eq. (21) differs. The contributions from the nontuning poles and the nondominant contributions from the tuning poles are conventional and would attain their peaks during the ground excitation or shortly thereafter. The dominant response from the tuning poles, on the other hand, is controlled by the energy transfer from the structure to the equipment through beating, which takes a relatively long time [1, 2], and the peak response of this contribution will occur substantially after the end of the ground motion.

The peak response of grossly nontuned systems, *i.e.*, where the equipment frequency is far from all structural frequencies, can be estimated in the same way. For light equipment, the structure poles are only slightly shifted from their location for the structure alone and additional poles due to the equipment occur close to those for the equipment alone:

$$p = -\beta\omega \pm i\omega \qquad (22)$$

The residues at the structure poles are as before. The residues at the equipment poles contribute a term similar to the nondominant term in the tuned case. The derivation is standard and the complete response for the equipment in the grossly nontuned case is given by

$$\ddot{u}(t) = \int_0^t \ddot{u}_g(\tau) \left\{ \sum_{m=1}^{N} \frac{C_r^m}{1-(\Omega_m/\omega)^2} \Omega_m e^{-B_m \Omega_m (t-\tau)} \sin\Omega_m (t-\tau) \right.$$

$$\left. + \sum_{m=1}^{N} \frac{C_r^m}{1-(\omega/\Omega_m)^2} \omega e^{-\beta\omega(t-\tau)} \sin\omega(t-\tau) \right\} d\tau \qquad (23)$$

APPROXIMATE RESPONSE SPECTRA RESULTS

The results given in the previous section can be applied to the design of equipment and equipment mounting. Typically, the information available to the designer is a ground shock spectrum. It was shown in references 1 and 2 that for the case of the undamped system, both tuned and nearly tuned, the peak acceleration in the equipment could be related to the response

spectrum of the ground motion. The present paper is concerned with the generalization of these results to the damped tuned or nontuned cases.

For the grossly nontuned system where the equipment frequency is far away from all structural frequencies and these are well separated, conventional summation methods, *e.g.*, the square root of the sum of the squares procedure applied to eq. (23), lead to an estimate for $|\ddot{u}|_{\max}$ in the form

$$|\ddot{u}|_{\max} = \left\{ \sum_{m=1}^{N} \left[\frac{C_r^m}{1-(\Omega_m/\omega)^2} S_A(\Omega_m, B_m) \right]^2 + \left[\sum_{m=1}^{N} \frac{C_r^m}{1-(\omega/\Omega_m)^2} \right]^2 S_A^2(\omega,\beta) \right\}^{1/2} \tag{24}$$

where $S_A(\Omega, B)$ is the acceleration ground shock spectrum for the ground motion $\ddot{u}_g$ evaluated at frequency Ω and damping factor B. Other responses can be estimated by substituting the appropriate ground shock spectra in the above result.

The approach for tuned undamped systems is described in detail in reference 1. This led to the following estimates of maximum acceleration and displacement

$$|\ddot{u}|_{\max} = \frac{S_A(\omega, 0)}{\gamma^{1/2}} \tag{25}$$

and

$$|u|_{\max} = \frac{S_D(\omega, 0)}{\gamma^{1/2}} \tag{26}$$

where $S_A(\omega,\beta)$, $S_D(\omega,\beta)$ are the acceleration and displacement ground shock spectra for frequency ω and damping factor β.

The simplicity of the result can be explained on physical grounds. In weakly coupled systems with the same frequency, the response of the system involves a perfect energy exchange between each component at a beat frequency much lower than the natural frequency of each component. The same phenomenon--a classical beat phenomenon--occurs here. The coupling is weak because the ratio of equipment mass to structure mass is small.

When a structure is subjected to a ground motion, the velocity imparted to the structure is mass independent and determined only by the ground motion. Thus, if the same ground motion were applied directly to tuned equipment, the same velocity would be transmitted to it. Kinetic energy, on the other hand, is proportional to the mass of the system excited; in equipment, that energy would be much smaller than in a structure. However, if the equipment were attached to a structure and the structure subjected to a ground motion, the kinetic energy imparted to the latter would be wholly transmitted to the equipment, if tuned, and the velocity imparted would be amplified by the reciprocal of the square root of the mass ratio. Damping is clearly important in this process because the energy transfer requires many cycles and much of the kinetic energy in a damped system could be dissipated before being transmitted. Depending on the parameters of the system, the response may exhibit underdamped beats as shown in Figure 2, critically damped beats, or overdamped beats. In order to demonstrate the physical and analytical basis of the method we consider the special case of a damped tuned system with $\gamma > (\beta - B_n)^2$. Other cases can be handled in exactly the same manner.

When $\xi = 0$ and $\gamma > (\beta - B_n)^2$ the contribution from the tuning poles in eq. (21) becomes

$$\ddot{u}(t) = -\frac{C_r^n \omega}{[\gamma-(\beta-B_n)^2]^{1/2}} \int_0^t \ddot{u}_g(\tau)\, [e^{-(\beta+B_n)\omega(t-\tau)/2} \cos\omega(t-\tau) \sin\eta(t-\tau)]\, d\tau \tag{27}$$

where

$$\eta = [\gamma - (\beta - B_n)^2]^{1/2}\omega/2$$

We note that the term in brackets in the integral represents a damped beat motion as shown in Figure 2 with a damping factor $(\beta + B_n)/2$ and a beat period T given by $2\pi/\eta$. When the term $\sin\eta(t-\tau)$ is expanded

$$\ddot{u}(t) = -\frac{C_r''\omega}{[\gamma-(\beta-B_n)^2]^{1/2}}\cos(\eta t-\theta)\left\{\left[\int_0^t \ddot{u}_g(\tau)e^{-(\beta+B_n)\omega(t-\tau)/2}\cos\omega(t-\tau)\cos\eta\tau\,d\tau\right]^2 + \left[\int_0^t \ddot{u}_g(\tau)e^{-(\beta+B_n)\omega(t-\tau)/2}\cos\omega(t-\tau)\sin\eta\tau\,d\tau\right]^2\right\}^{1/2} \tag{28}$$

where

$$\theta = \tan^{-1}\left\{-\frac{\int_0^t \ddot{u}_g(\tau)\cos\eta\tau\, e^{-(\beta+B_n)\omega(t-\tau)/2}\cos\omega(t-\tau)\,d\tau}{\int_0^t \ddot{u}_g(\tau)\sin\eta\tau\, e^{-(\beta+B_n)\omega(t-\tau)/2}\cos\omega(t-\tau)\,d\tau}\right\} \tag{29}$$

We consider $\eta t_1 = 2\pi t_1/T << 1$, where t_1 is the duration of ground motion and T is the beat period of the system. Then the first integral in Eq. (28) can be approximated by

$$\int_0^t \ddot{u}_g(\tau)\, e^{-(\beta+B_n)\omega(t-\tau)/2}\cos\omega(t-\tau)\,d\tau$$

and the second neglected since $\sin\eta t$ will be bounded by $\eta t_1 << 1$ and $\ddot{u}_g = 0$ for $t > t_1$. Thus, we take

$$\ddot{u}(t) = -\frac{C_r''\omega\sin\eta t}{[\gamma-(\beta-B_n)^2]^{1/2}}\int_0^t \ddot{u}_g(\tau)\, e^{-(\beta+B_n)\omega(t-\tau)/2}\cos\omega(t-\tau)\,d\tau \tag{30}$$

When the parameters $\gamma^{1/2}$, β, and B_n are small, this result may be interpreted in the following way: for $t > t_1$, the above expression can be written in the form

$$\ddot{u}(t) = -\frac{C_r''\omega^2\sin\eta t}{2\eta}\, e^{-(\beta+B_n)\omega t/2} R\cos(\omega t-\psi)$$

where

$$R = (A_1^2 + A_2^2)^{1/2}$$

with

$$A_1 = \int_0^{t_1} \ddot{u}_g(t)\, e^{+(\beta+B_n)\omega t/2}\cos\omega t\,dt$$

$$A_2 = \int_0^{t_1} \ddot{u}_g(t)\, e^{+(\beta+B_n)\omega t/2}\sin\omega t\,dt$$

and

$$\psi = \tan^{-1}(A_2/A_1)$$

The response indicated by the above is illustrated in Fig. 5. In the above, the terms R and ψ are constants independent of t for $t > t_1$, and $R\cos(\omega t-\psi)$ is a rapidly varying function of time. The term

$$\frac{\omega^2}{2\eta}e^{-(\beta+B_n)\omega t/2}\sin\eta t$$

is a slowly varying envelope curve whose maximum value must be determined. The maximum value of this envelope curve is attained at time t^*, expressed by

$$\tan\eta t^* = \frac{2\eta}{(\beta+B_n)\omega} \tag{31}$$

The value of t^* is thus

$$t^* = \arctan[2\eta/\omega(\beta+B_n)]/\eta \tag{32}$$

For lightly damped systems and light equipment, in general $t^* >> t_1$. The values of $\sin\eta t$ and $\exp[-(\beta+B_n)\omega t/2]$ when the maximum of the envelope is achieved are

$$\sin\eta t^* = \frac{\eta}{[\eta^2+(\beta+B_n)^2\omega^2/4]^{1/2}}$$

$$e^{-(\beta+B_n)\omega t^*/2} = e^{-\kappa} \tag{33}$$

where

$$\kappa = (\arctan\zeta)/\zeta \tag{34}$$

$$\zeta = [\gamma-(\beta-B_n)^2]^{1/2}/(\beta+B_n) \tag{35}$$

It follows that

$$|\ddot{u}|_{\max} = |\ddot{u}(t^*)| = \left\{\frac{C_r^n\omega}{[\gamma-(\beta-B_n)^2]^{1/2}}|\sin\eta t^*|e^{-\kappa}\right\}$$
$$\left\{e^{(\beta+B_n)\omega\eta t^*/2}\left|\int_0^{t^*}\ddot{u}_g(\tau)e^{-(\beta+B_n)\omega(t^*-\tau)/2}\cos\omega(t^*-\tau)\,d\tau\right|\right\} \tag{36}$$

In order that this estimate of peak acceleration be useful for design purposes, it is necessary that the second factor in braces be interpreted in terms of a ground response spectrum. To this end, we recognize that the integral is, to the order of $\beta+B_n$, the relative velocity response history evaluated at time t^* of a lightly damped single-degree-of-freedom oscillator of frequency ω and damping factor $(\beta+B_n)/2$ subjected to the ground acceleration $\ddot{u}_g(t)$. At some time $\tilde{t}$ during the ground motion or shortly after it ceases (so that $\tilde{t} << t^*$), the absolute value of the relative velocity will attain its global maximum, denoted as $|v(\tilde{t})|$. The relative velocity response at t^*, denoted as $v(t^*)$, can be thought of as that which would occur in a single-degree-of-freedom system (subjected to the ground acceleration $\ddot{u}_g(t)$) as a consequence of free vibration beginning at time $\hat{t}(>t_1)$ when the absolute value of the relative velocity of the oscillator attains its first local maximum, $|v(\hat{t})|$, after the end of the ground motion. This instant of time $\hat{t}$ is equal to $\tilde{t}$ if $\tilde{t}$ occurs after the end of the ground motion; otherwise, $\hat{t} > \tilde{t}$. In any event, $\hat{t} << t^*$. Thus, we can write

$$|v(t^*)| = |v(\hat{t})|e^{-(\beta+B_n)\omega(t^*-\hat{t})/2}|\cos\omega(t^*-\hat{t})| \tag{37}$$

where $|v(\hat{t})| \leqslant |v(\tilde{t})|$. It then follows that

$$|v(t^*)| \equiv |\int_0^{t^*} \ddot{u}_g(\tau) e^{-(\beta+B_n)\omega(t^*-\tau)/2} \cos\omega(t^*-\tau)\, d\tau|$$

$$= |v(\hat{t})| e^{-(\beta+B_n)\omega(t^*-\hat{t})/2} |\cos\omega(t^*-\hat{t})|$$

$$\leqslant |v(\tilde{t})| e^{-(\beta+B_n)\omega t^*(1-\hat{t}/t^*)/2}$$

$$\approx |v(\tilde{t})| e^{-(\beta+B_n)\omega t^*/2} \tag{38}$$

since $\hat{t}/t^* << 1$. From this we obtain the approximate result

$$|v(\tilde{t})| \approx e^{(\beta+B_n)\omega t^*/2} |\int_0^{t^*} \ddot{u}_g(\tau) e^{-(\beta+B_n)\omega(t^*-\tau)/2} \cos\omega(t^*-\tau)\, d\tau| \tag{39}$$

We recognize, however, that to the order of β and B_n, $|v(\tilde{t})|$ is very nearly the pseudo-velocity response spectrum $S_V(\omega, (\beta+B_n)/2)$ for a lightly damped single-degree-of-freedom oscillator of frequency ω and damping factor $(\beta+B_n)/2$ subjected to the ground acceleration $\ddot{u}_g(t)$. Thus, an estimate of the maximum equipment acceleration is

$$|\ddot{u}|_{\max} = \frac{|C_r^n|\omega|\sin\eta t^*|e^{-\kappa}}{[\gamma-(\beta-B_n)^2]^{1/2}} S_V(\omega, \frac{\beta+B_n}{2})$$

With the value of $\sin\eta t^*$ from Eq. (33), we obtain the final estimate as

$$|\ddot{u}|_{\max} = \frac{|C_r^n|e^{-\kappa}\omega S_V(\omega, \frac{\beta+B_n}{2})}{(\gamma+4\beta B_n)^{1/2}} \tag{40}$$

Recalling that

$$\omega S_V = S_A = \omega^2 S_D$$

this estimate can be written in the alternative form

$$|\ddot{u}(t)|_{\max} = \frac{|C_r^n|e^{-\kappa}}{(\gamma+4\beta B_n)^{1/2}} S_A(\omega, \frac{\beta+B_n}{2}) \tag{41}$$

Estimates for other cases of damped tuned systems, *e.g.*, where $\gamma < (\beta - B_n)^2$, $\gamma = (\beta - B_n)^2$ and $\gamma = B_n^2$, $\beta = 0$, were developed in [11]; precisely the same result was obtained.

For a slightly nontuned system, the dominant contribution to the response is given by

$$\ddot{u}(t) = -\frac{C_r^n \omega}{(\lambda^2+\mu^2)} \int_0^t \ddot{u}_g(\tau)\, e^{-(\beta+B_n)\omega(t-\tau)/2}$$

$$\Bigg\{ -\lambda \sinh\frac{\mu}{2}\omega(t-\tau) \cos\frac{\lambda}{2}\omega(t-\tau) \sin(1+\frac{\xi}{2})\omega(t-\tau)$$

$$+\lambda \cosh\frac{\mu}{2}\omega(t-\tau) \sin\frac{\lambda}{2}\omega(t-\tau) \cos(1+\frac{\xi}{2})\omega(t-\tau)$$

$$+\mu \sinh\frac{\mu}{2}\omega(t-\tau) \cos\frac{\lambda}{2}\omega(t-\tau) \cos(1+\frac{\xi}{2})\omega(t-\tau)$$

$$+\mu \cosh\frac{\mu}{2}\omega(t-\tau) \sin\frac{\gamma}{2}\omega(t-\tau) \sin(1+\frac{\xi}{2})\omega(t-\tau) \Bigg\} d\tau \qquad (42)$$

where λ and μ are defined in terms of ξ, γ, β, and B_n in Eqs. (16) and (17).

The reduction of this general form to the required amplified response spectrum form is given in detail in reference 11 and takes the form

$$|\ddot{u}|_{\max} = \frac{|C_r^n| e^{-\kappa}}{(\gamma+\xi^2+4\beta B_n)^{1/2}} S_A\left(\frac{\omega+\Omega_n}{2}, \frac{\beta+B_n}{2}\right) \qquad (43)$$

where

$$\kappa = (arctan\zeta)/\zeta \qquad (44)$$

$$\zeta = \left[\gamma+\xi^2-(\beta-B_n)^2\right]^{1/2}/(\beta+B_n) \qquad (45)$$

The contribution to the equipment acceleration from the other modes for which no interaction need be considered is quite standard. Using eqs. (8) and (9) in conjunction with the square root of the sum of squares procedure the estimate of the maximum acceleration in the early peak, which will occur during the ground motion, is given by

$$|\ddot{u}|_{\max} = \left\{ \sum_{\substack{m=1\\ m\neq n}}^{N} \left[\frac{C_r^m}{1-(\Omega_m/\omega)^2} S_A(\Omega_m, B_m)\right]^2 + \left[\sum_{\substack{m=1\\ m\neq n}}^{N} \frac{C_r^m}{1-(\omega/\Omega_m)^2}\right]^2 S_A^2(\omega,\beta) \right\}^{1/2} \qquad (46)$$

The peak value given by eq. (46) occurs early after only a few cycles of equipment motion and that given by eq. (43) occurs late and thus in principle they should not be taken together. However, we have recently completed extensive numerical and physical experimentation the results of which appear to suggest ways in which the results can be conveniently applied to equipment mounting design. The physical experiments were carried out at the Earthquake Simulator Laboratory of the Earthquake Engineering Research Center, University of California, Berkeley, and involved the use of a three-story steel frame model structure (Figure 3) around one-third full scale. The response of the steel frame model is roughly that of a shear building with a natural frequencies at around 2 Hz, 8 Hz, and 15 Hz. The total weight of the model was 39,500 lbs and its height was about 20 feet. Three single-degree-of-freedom oscillators (Figure 4) were attached to the concrete blocks at the second and third floors to simulate equipment in a primary structure. The test structure was instrumented to measure displacements and accelerations at each floor and acceleration of the oscillators. The mechanical oscillators, each weighing about 20 lbs, were constructed to correspond to the first three natural frequencies of the model structure.

The first three natural frequencies of the frame were determined accurately and the oscillators were tuned to the structural frequencies. They were then bolted to the floors of the model structure. A variety of ground shock table motions was applied to the model, some of

which were short duration approximate square waves and some of which were longer duration and more erratic. Figure 5 shows the time history of acceleration in a typical run for a short duration single displacement pulse. This produced peak table acceleration pulses around ± 0.16g, separated by about 1 sec. The peak responses in the tuned oscillators were 2.0g, 3.5g, and 0.6g in the first, second, and third mode oscillators, respectively. On Figure 5, the top trace shows the input table acceleration, the second the response of the third floor of the structure, and the next three the accelerations of the three tuned oscillators. The very large magnification of acceleration experienced by the oscillators is immediately obvious. The beat phenomenon is clear. The peak acceleration in the first two oscillators is achieved considerably after the peak acceleration in the input. These two oscillators are obviously responding at the coupled frequencies governed by equipment-structure interaction.

The response of the third oscillator is quite different; a highly irregular pattern appears and local maxima occur during as well as after the excitation. Fourier transforms of the acceleration time histories of the three oscillators have been taken to clarify this response (Figure 6). It is clear from these transforms that the first mode oscillator responded only at the first mode frequency, around 2 Hz; the second mode oscillator responded predominantly at the second mode frequency, around 8 Hz, with a small contribution from the first mode. The dominant contribution to the response of the third mode oscillator was from frequencies around the third mode frequency of 15 Hz, but significant contributions also appeared from the lower modes. Thus, the pure damped beat response predicted by the theory was complicated by contributions from the lower modes.

These observations are supported by the numerical experiments and have implications for the application of the theoretical results to design. First, only the structural modes with frequencies up to and around that of the equipment under consideration need be considered and, second, the late peak acceleration given by eq. (43) should be summed with the early peak acceleration given by eq. (46) by an appropriate summation rule. For instance, if the conventional square root of the sum of the squares procedure is used, the estimate is

$$|\ddot{u}|_{\max} = \left\{ \sum_{\substack{m=1 \\ m\neq n}}^{N} \left[\frac{C_r^m}{1-(\Omega_m/\omega)^2} S_A(\Omega_m, B_m) \right]^2 + \left[\sum_{\substack{m=1 \\ m\neq n}}^{N} \frac{C_r^m}{1-(\omega/\Omega_m)^2} \right]^2 S_A^2(\omega,\beta) + \left[\frac{|C_r^n| e^{-\kappa}}{(\gamma+\xi^2+4\beta B_n)^{1/2}} S_A\left[\frac{\omega+\Omega_n}{2}, \frac{\beta+B_n}{2} \right] \right]^2 \right\}^{1/2}$$

If the late peak occurs well after the peak of the excitation (as in the first mode oscillator) it will be the dominant term and the result will be nearly the same as if it alone were considered. On the other hand, if the late peak occurs during the excitation (as in the third mode oscillator) then it should be superposed with the other modal contributions which may occur around the same time and this will be effected by the formula.

NUMERICAL EXPERIMENTS

In addition to the physical experiments, we have performed numerical experiments using a standard structural analysis program, TABS [12], to compute by a variety of methods the response of a light appendage in a building. The structure used in these numerical experiments was a ten-story reinforced concrete frame building described and used in reference 13; the appendage was a single-degree-of-freedom oscillator attached to the top floor and had a mass which gave a mass ratio of 0.001 compared to the modal mass of the first mode of the building. The response of the appendage was calculated using TABS to evaluate the eleven-degree-of-freedom system which consisted of the structure and the appendage. The program has the capability of computing natural frequencies and mode shapes and then can either compute a

.ime history in each mode (by a method which is exact for piecewise-linear acceleration records) and sums these for the resultant response, or can determine the maximum response in each mode by a spectrum method and estimate the maximum response by the square root of the sum of the squares procedure.

The response of the appendage as a function of appendage frequency (maintaining the same mass ratio) was calculated by both methods for a variety of ground motions. Two cases of damping were considered: undamped and 2% of critical damping in each mode. Typical results for a very short duration ground motion for both methods of computation are shown in Figure 7 for the undamped case and in Figure 8 for the damped case. The TABS time-history results, which should be exact, are shown by solid dots and the TABS spectrum (SRSS) results are shown by triangles. The tuned system estimate based on eq. (47) is shown as a solid curve and the nontuned system estimate based on eq. (24) is indicated by squares on the same figures.

It is clear from the diagrams that for both damped and undamped cases the tuned estimate compares well with the nontuned estimate in the regions of gross nontuning. (This is probably due to the even spacing of the natural frequencies of the structure in this case; thus, the tuned estimate could be used for structures with relatively evenly spaced natural frequencies except possibly for frequencies well below the first or much above the highest.) Further, the tuned estimate compares well with the time-history calculation over the entire range of appendage frequencies considered, which encompasses the first three modes of the structure. This is surprising in view of the fact that the spectra used in these numerical experiments are jagged spectra and not the smoothed design spectra for which the approach was developed. A further result is that the TABS spectrum (SRSS) estimate is very poor near tuning for all modes considered, but is very good for grossly nontuned cases. This is not surprising since it is well known that SRSS is not accurate for closely spaced modes which naturally occur when a light appendage or equipment has a frequency near one of the structure's frequencies. In this case, the standard approach is to use the NRL method, which for a two-degree-of-freedom system is equivalent to the absolute sum method. From Figure 7, the TABS time-history result for the undamped case and when the appendage is tuned to the first mode is 78.2. The TABS spectrum (SRSS) result is 55.1 and if the absolute sum method is used the result is 77.9. The estimate given by the formula (47) for the tuned case is 78.1. However, the TABS spectrum (SRSS) result greatly overestimates the response in the damped case as shown in Figure 8. The TABS time-history computation is 20.2; the TABS spectrum (SRSS) result is 50.2. (If the absolute sum method is used, the result is 71.0.) The tuned result from eq. (47) is 19.4.

The physical explanation for this overestimation by the SRSS and absolute sum methods in the damped case is that the beat phenomenon involves an energy transfer between the two elements of the system. In the case of an undamped, perfectly tuned system, the energy transfer is complete, *i.e.*, maximum response in one element is accompanied by zero response in the other. This energy transfer takes time; the beat period that controls the energy transfer is inversely proportional to the difference between the two closely spaced frequencies. The phenomenon can be interpreted geometrically in terms of the eigenvectors of the modes. The components of these modes can be thought of as vectors in a generalized state space. The components that represent the appendage are initially 180 degrees out of phase and those of the structure are in phase. As the motion continues, the equipment components rotate and eventually align, while the structural components become out of phase. As the vectors rotate, the resultant for the equipment increases and if the system is undamped, attains a maximum when they are aligned. In the damped case, the peak values of the vectors will first increase and then diminish as they rotate and the resultant will achieve its maximum before they line up. Thus, it is important in the damped case to determine at which point the maximum of the resultant occurs. In terms of this analogy, the square root of the sum of the squares procedure assumes that the peak values of both of the vectors are attained simultaneously and occur when they are at 90 degrees. The absolute sum method assumes that this occurs when they are lined up. The method described here evaluates the position at which the resultant actually attains its maximum.

The overestimation at tuning of the SRSS or absolute sum methods for cases with damping can be quantified by considering only the two-degree-of-freedom system equivalent to the tuned appendage-structure system. The three estimates of peak acceleration based on the SRSS rule, the absolute method, and our formula, eq. (47), respectively, are given by

$$|\ddot{x}|_{\max}^{srss} = \frac{1}{\sqrt{2\gamma}} S_A(\omega,\beta)$$

$$|\ddot{x}|_{\max}^{asm} = \frac{1}{\sqrt{\gamma}} S_A(\omega,\beta)$$

$$|\ddot{x}|_{\max}^{new} = \frac{e^{-\kappa}}{\sqrt{\gamma+4\beta^2}} S_A(\omega,\beta)$$

where for simplicity B_n is taken to be the same as β. We note immediately that if $\beta \to 0$, the square root of the sum of the squares estimate will be low by a factor of $\sqrt{2}$; the absolute sum estimate and the new estimate are in this case identical. However, the estimates differ for nonzero damping. We introduce an overestimation ratio

$$R = \frac{|\ddot{x}|_{\max}^{asm}}{|\ddot{x}|_{\max}^{new}} = (1 + 4\beta^2/\gamma)^{1/2} e^{\kappa} \tag{48}$$

Since κ is given by

$$\kappa = arctan\ \frac{\gamma^{1/2}/2\beta}{\gamma^{1/2}/2\beta}$$

the overestimation ratio can be expressed in terms of the single parameter $\gamma^{1/2}/2\beta$, in terms of which a plot of eq. (48) is given in Figure 9. The result can also be shown for several values of γ as a function of β; the resulting curves are shown in Figure 10. Clearly, for all values of γ, the overestimation parameter and thus the conservatism of the conventional method steadily increase with β. If we fix γ and decrease β such that

$$\gamma^{1/2}/2\beta \to \infty$$

we find that

$$\kappa \to \frac{\pi}{2}\ \frac{2\beta}{\gamma^{1/2}}$$

and

$$R \to (1 + \frac{2\beta^2}{\gamma}) e^{\frac{\pi}{2}(2\beta/\gamma^{1/2})}$$

which indicates that $R > 1$ for all nonzero β.

On the other hand, if we fix β and decrease γ such that $\gamma^{1/2}/2\beta \to 0$, we find that for light appendage cases, which correspond to very closely spaced modes, the response can be greatly overestimated.

It is interesting to compare these results to those from the numerical experiments on the multidegree-of-freedom structure. The absolute sum method estimate for the 2% damped case (as shown in Figure 8) is 71.0. The new estimate, from eq. (47), is 19.4, and the time-history result is 20.2. The overestimate of the absolute sum method is thus 71.0/20.2 = 3.52; for γ = 0.001 and β = 0.02, the value of R is 3.76. This indicates the excellent applicability of the simple analysis based on the one-degree-of-freedom structure to the multidegree-of-freedom structure. The reason for this is the dominance of the two closely spaced modes in the composite system when tuned.

OPTIMUM DAMPING

The solution for the nearly tuned two-degree-of-freedom system given in an earlier section depends on a considerable number of parameters and it is difficult to interpret the influence of individual terms. The damping can obviously dominate the solution if $4\beta B >> \gamma + \xi^2$, but it is also clear that the difference between damping factors in the structure and the equipment can play a significant role in the term ζ and thus in the amplification factor. Thus, it is appropriate to ask the question: for what values of β and B, given fixed values of γ, ξ^2, and $\beta + B$, is the amplification A minimized? To this end, we denote

$$\gamma + \xi^2 = a^2$$

and

$$\beta + B = c$$

and take a and c as fixed positive numbers.

We also note that in the case where $\gamma + \xi^2 > (\beta - B)^2$, the denominator $(\gamma + \xi^2 + 4\beta B)^{1/2}$ can be written in the form $c(1+\zeta)^{1/2}$. The amplification function A becomes

$$A = \frac{e^{-\kappa(\zeta)}}{c(1+\zeta^2)^{1/2}}$$

Considered as a function of ζ, A has extremal values when

$$c\,\frac{dA}{d\zeta} = e^{-\kappa}\left[-\frac{1}{(1+\zeta^2)^{1/2}}\frac{d\kappa}{d\zeta} - \frac{\zeta}{(1+\zeta^2)^{\frac{3}{2}}}\right] = 0$$

Now

$$\frac{d\kappa}{d\zeta} = -\frac{1}{\zeta^2}\,arctan\,\zeta + \frac{1}{\zeta}\,\frac{1}{1+\zeta^2}$$

and it follows that

$$c\,\frac{dA}{d\zeta} = \frac{e^{-\kappa}}{\zeta(1+\zeta^2)^{1/2}}\left[\frac{arctan\,\zeta}{\zeta} - \frac{1}{1+\zeta^2} - \frac{\zeta^2}{1+\zeta^2}\right] = \frac{e^{-\kappa}}{\zeta(1+\zeta^2)^{1/2}}\left(\frac{arctan\,\zeta}{\zeta} - 1\right)$$

Thus, $dA/d\zeta \rightarrow 0$ when $\zeta \rightarrow \infty$. When $\zeta \rightarrow 0$, we have

$$c\,\frac{dA}{d\zeta} \rightarrow \frac{e^{-1}}{\zeta}\left(1 - \frac{1}{3}\zeta^2 - 1\right) \rightarrow 0$$

There are no other zeroes of $dA/d\zeta$. We note that as $\zeta \rightarrow 0$, $A(\zeta)$ has the form

$$A(\zeta) = \frac{e^{-1}}{c}\left(1 - \frac{\zeta^2}{6}\right)$$

Thus, the function $A(\zeta)$ is a monotonically decreasing function of ζ. To minimize A, then, it is necessary to maximize ζ and this is achieved obviously when $\beta = B$. The conclusion is therefore that when

$$\gamma + \xi^2 > (\beta - B)^2$$

the optimal damping for fixed $\beta + B$ is $\beta = B$. The value of A for this case is given by

$$A_{\min} = \frac{e^{-\kappa}}{(\gamma + \xi^2 + c^2)^{1/2}}$$

with $\kappa = arctan\,\zeta/\zeta$ and $\zeta = (\gamma + \xi^2)^{1/2}/c$. We note that this value is always less than $e^{-\kappa}/c$. If c is sufficiently large, it is also possible to have

$$\gamma + \xi^2 < (\beta - B)^2$$

and in this case the amplification factor takes the form

$$A = \frac{e^{-\kappa}}{(\gamma+\xi^2+4\beta B)^{1/2}}$$

with $\kappa = \mathit{arctanh}\,\zeta/\zeta$, where now

$$\zeta = \frac{((\beta-B)^2-\gamma-\xi^2)^{1/2}}{\zeta}$$

In this case the denominator

$$(\gamma + \xi^2 + 4\beta B)^{1/2} = c\,(1-\zeta^2)^{1/2}$$

Thus,

$$c\,A\,(\zeta) = \frac{e^{-\kappa(\zeta)}}{(1-\zeta^2)^{1/2}}$$

Using the well-known expression for *arctanh* in terms of ln, we have

$$A = \frac{1}{c}\left(\frac{(1-\zeta)^{\frac{(1-\zeta)}{\zeta}}}{(1+\zeta)^{\frac{(1+\zeta)}{\zeta}}}\right)^{1/2}$$

and

$$\ln(cA) = \frac{1}{2\zeta}\{(1-\zeta)\ln(1-\zeta) - (1+\zeta)\ln(1+\zeta)\}$$

Thus

$$\frac{2d(\ln cA)}{d\zeta} = \frac{1}{\zeta^2}(-\ln(1-\zeta) + \ln(1+\zeta) - 2\zeta)$$

Thus, $dA/d\zeta \rightarrow 0$ when $\ln[(1+\zeta)/(1-\zeta)] = 2\zeta$ if $A \neq \infty$. This has only one solution, namely $\zeta = 0$.

The function $cA(\zeta)$ has two parts, $e^{-\kappa}$ and $1/(1-\zeta^2)^{1/2}$. When $\zeta \rightarrow 0$, $e^{-\kappa} \rightarrow e^{-1}(1-1/3\,\zeta^2)$, but $1/(1-\zeta^2)^{1/2} \rightarrow 1+1/6\,\zeta^2$. Thus, $cA \rightarrow 1+1/6\,\zeta^2$ for small ζ and approaches a limiting value as $\zeta \rightarrow 1$; the limit of A as $\zeta \rightarrow 1$ is given by

$$\lim_{\zeta\rightarrow 1} cA = \lim_{\zeta\rightarrow 1}\left(\frac{(1-\zeta)^{\frac{1-\zeta}{\zeta}}}{(1+\zeta)^{\frac{1+\zeta}{\zeta}}}\right)^{1/2} = \frac{1}{2}$$

Since the limit of cA as $\zeta \rightarrow 0$ is e^{-1} and $dA/d\zeta = 0$ only at $\zeta = 0$, the function increases monotonically from e^{-1} at $\zeta = 0$, to 1/2 at $\zeta = 1$. The minimum value of A is achieved here by making ζ as small as possible. If β and B are such that $(\gamma+\xi^2)$ can be equal to $(\beta-B)^2$, then the minimum is at $\zeta = 0$ and has the value e^{-1}/c, and β and B are given by

$$\beta = \frac{1}{2}(c \pm (\gamma + \xi^2)^{1/2})$$

$$B = \frac{1}{2}(c + (\gamma + \xi^2)^{1/2})$$

This requires that $c > (\gamma+\xi^2)^{1/2}$ in order to have realistic damping, but in any case the analysis of the previous case, $\gamma+\xi^2 > (\beta-B)^2$, gives a minimum for A which is always less than e^{-1}/c.

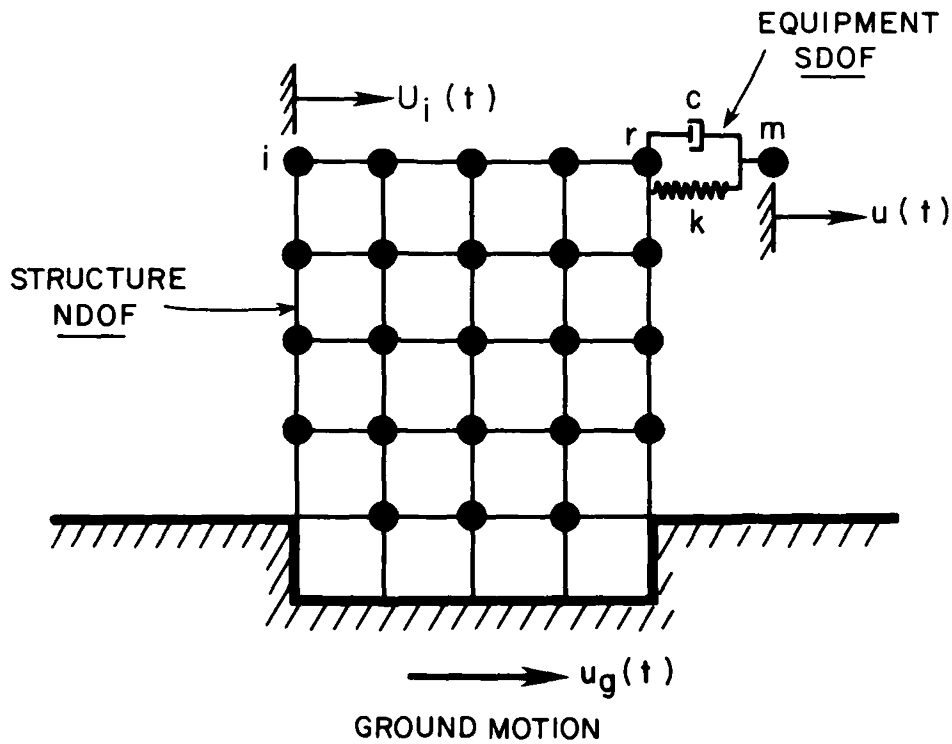
It follows then that in all cases the optimum choice of β and B is $\beta = B$.

CONCLUDING REMARKS

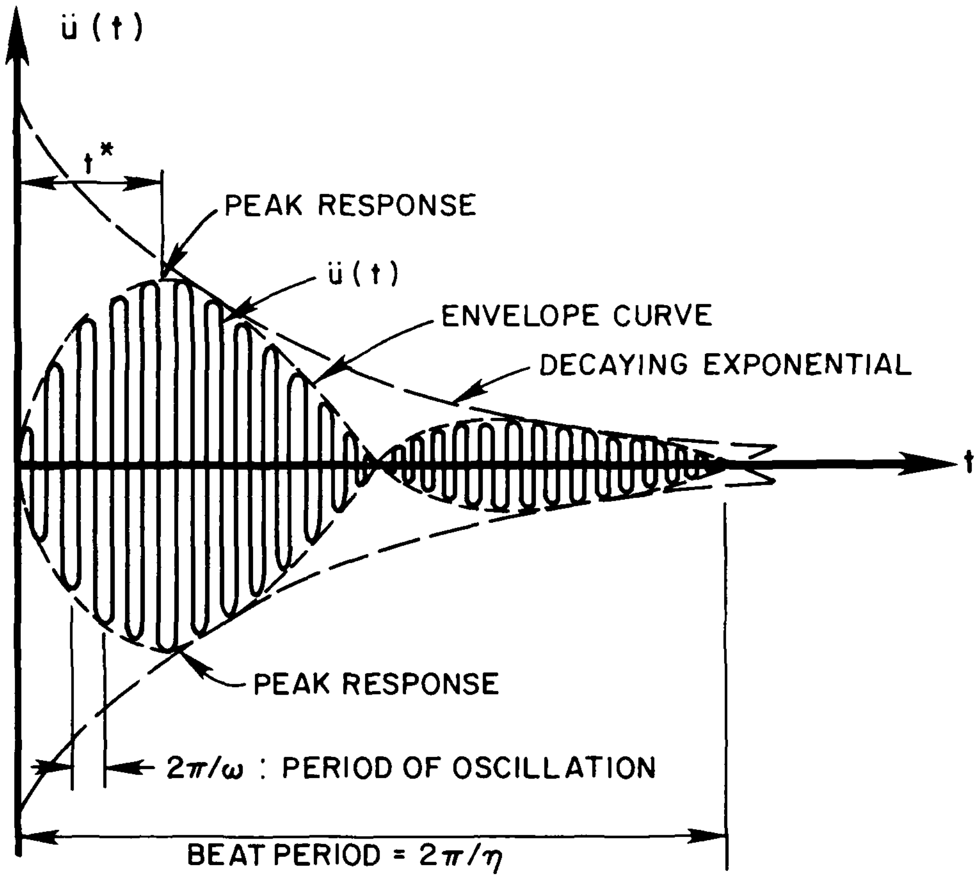
The advantages of the estimation technique developed in this paper are its simplicity and adaptability for practical application. A great deal of computational effort is avoided since time-history analyses need not be performed. The equipment and structure need not be analyzed as an $N+1$-degree-of-freedom system either by modal or matrix-time-marching methods, and errors in estimates of peak response due to the possible unreliability of numerical time integration schemes, or to uncertainty as to the appropriate procedure for summing the contributions of the two closely spaced modes, are thereby avoided. For tuned and nearly tuned systems, the method accounts for the important effect of equipment-structure interaction, which effect is completely neglected in the floor spectrum method. The method advanced here does not require that new information be generated. Data available from the building design alone, the equipment design alone, and the response spectra are used. The estimates of peak response have been obtained by rational analysis and are easily evaluated and conveniently used during the design process.

There are a number of obvious extensions of the analysis on which these simple estimates are based which could be of significance to DNA problems. The method which in effect introduces a transfer function between the induced ground motion and the response of the equipment avoids the explicit assumption of the support motion. For example, equipment such as a missle in a protective structure and suspended in this structure at several points could be treated by this approach. In the standard approach to multiple support response spectrum analysis of such a system the motion of each support would be calculated as if the suspended system were not present and this support motion applied to the suspended item. To use a response spectra method for such a system it would be necessary either to envelop the response spectra of the different support points or else to assume that each support point moves independently of the others and superpose the responses by a summation rule. This must provide a highly conservative prediction since it ignores the fact that all support motions are produced by a single input. It further cannot take into account closely spaced modes in the suspended item and the protective structure and this will provide even more conservatism as outlined in the paper. A straightforward extension of the present approach will provide response spectra estimates of the motion of the suspended item in terms of the ground motion utilizing the transfer functions for the support points, but not explicitly calculating their motion. Work on this is continuing.

A further extension of the analysis which is possible and could apply to DNA problems is to very long duration inputs which are known only in a statistical sense. Here, it is not possible to exploit the beat phenomenon and the consequent late peak to obtain the response spectrum result, but the transfer function result used as a point of departure still holds. Using this and the other assumptions of the analysis it will be possible to obtain statistical estimates of peak equipment response in terms of amplification factors and the statistics of the input ground motion. This could be a particularly valuable approach to the problem of a transporter vehicle carrying a sensitive item of equipment over a road or track with a statistically specified surface roughness.



Equipment-Structure System



Equipment Response History in the Case of Damped Beats

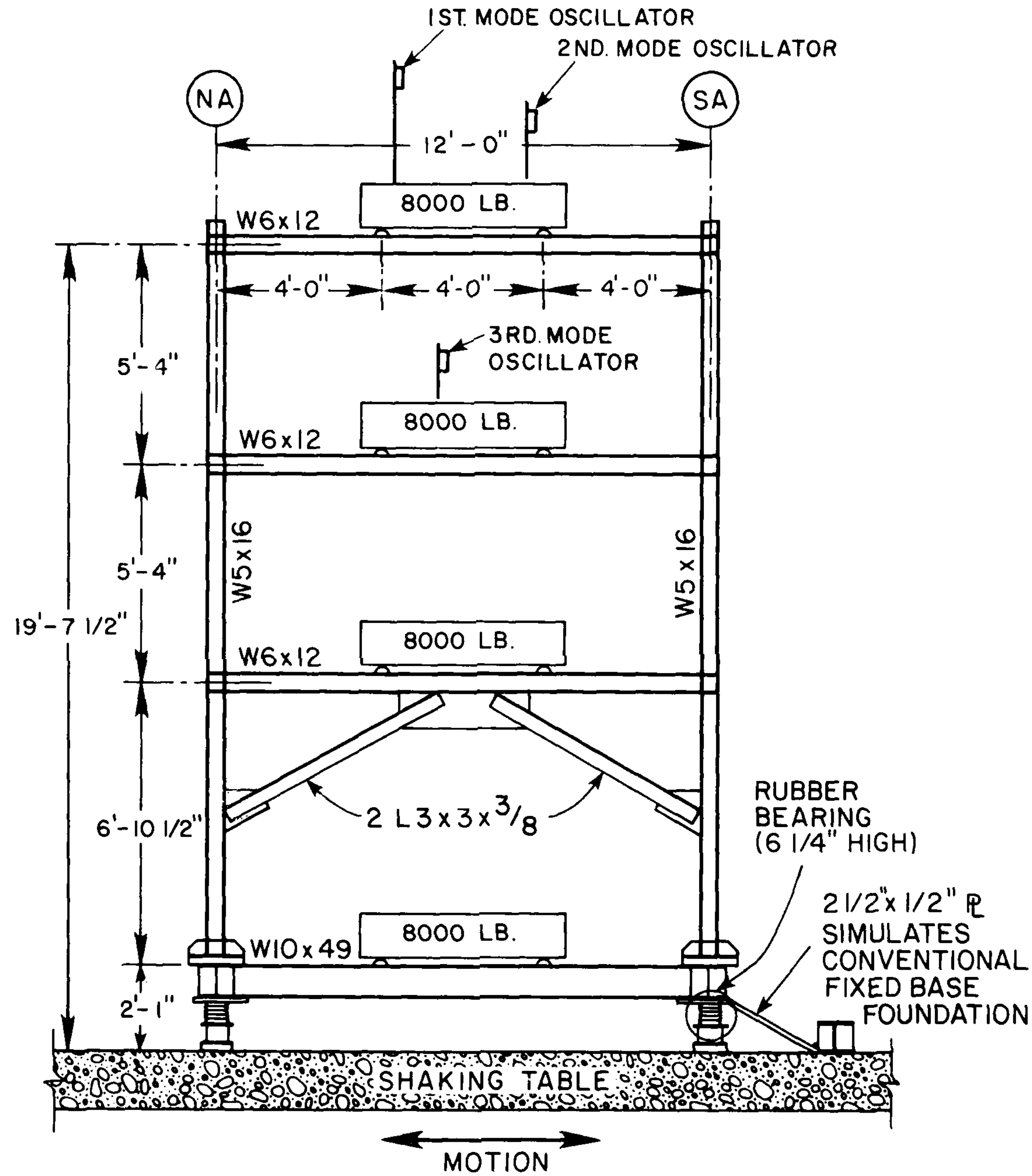


Diagram of Oscillators Attached to Model

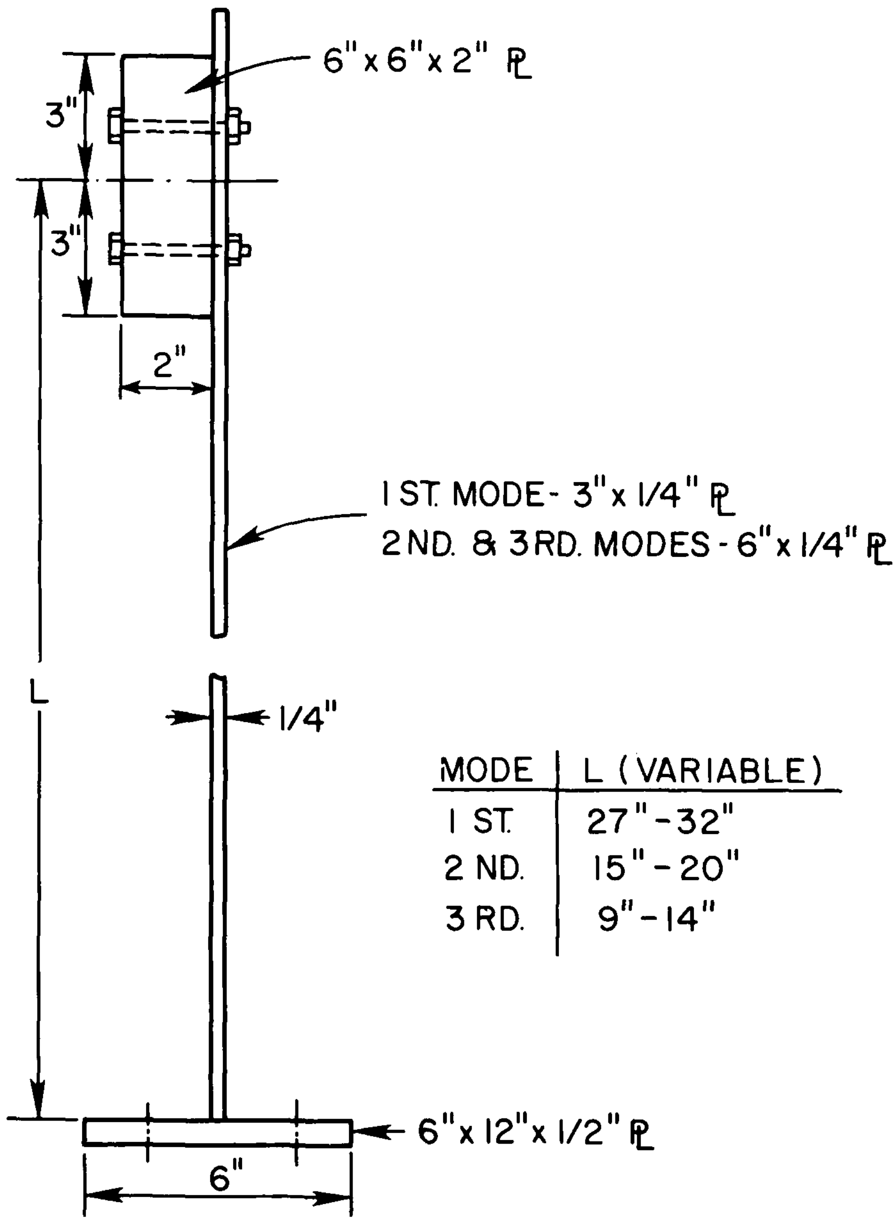
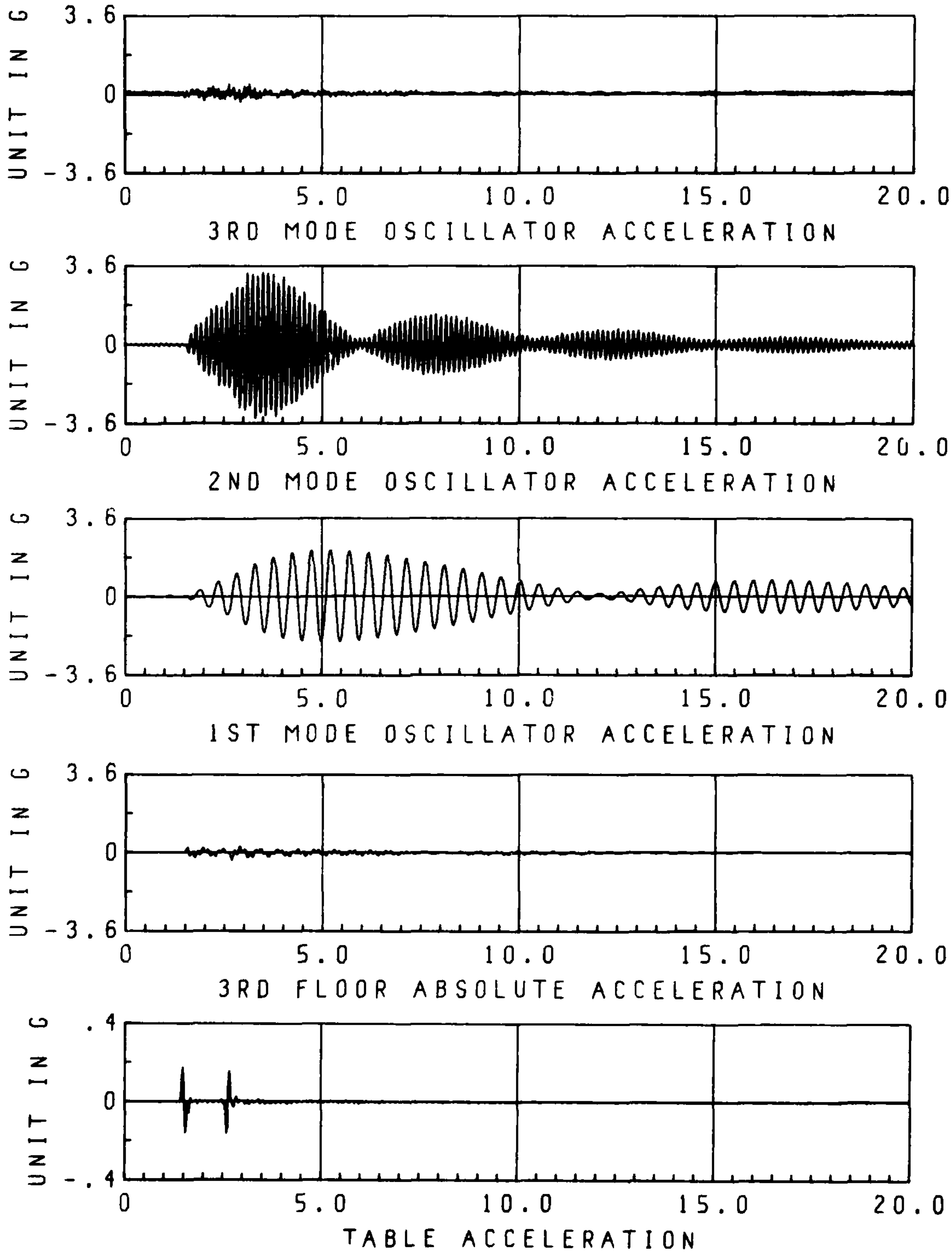
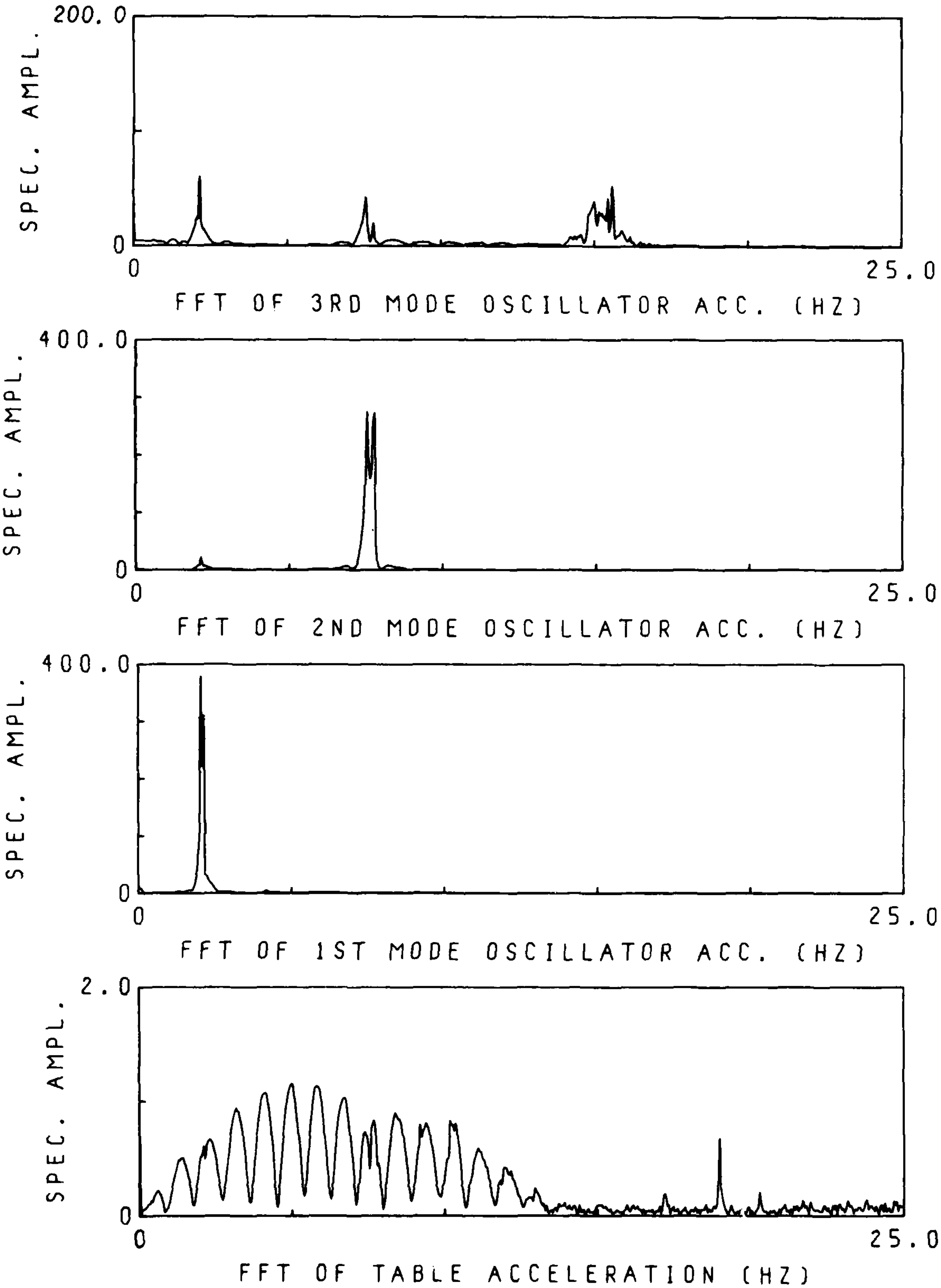


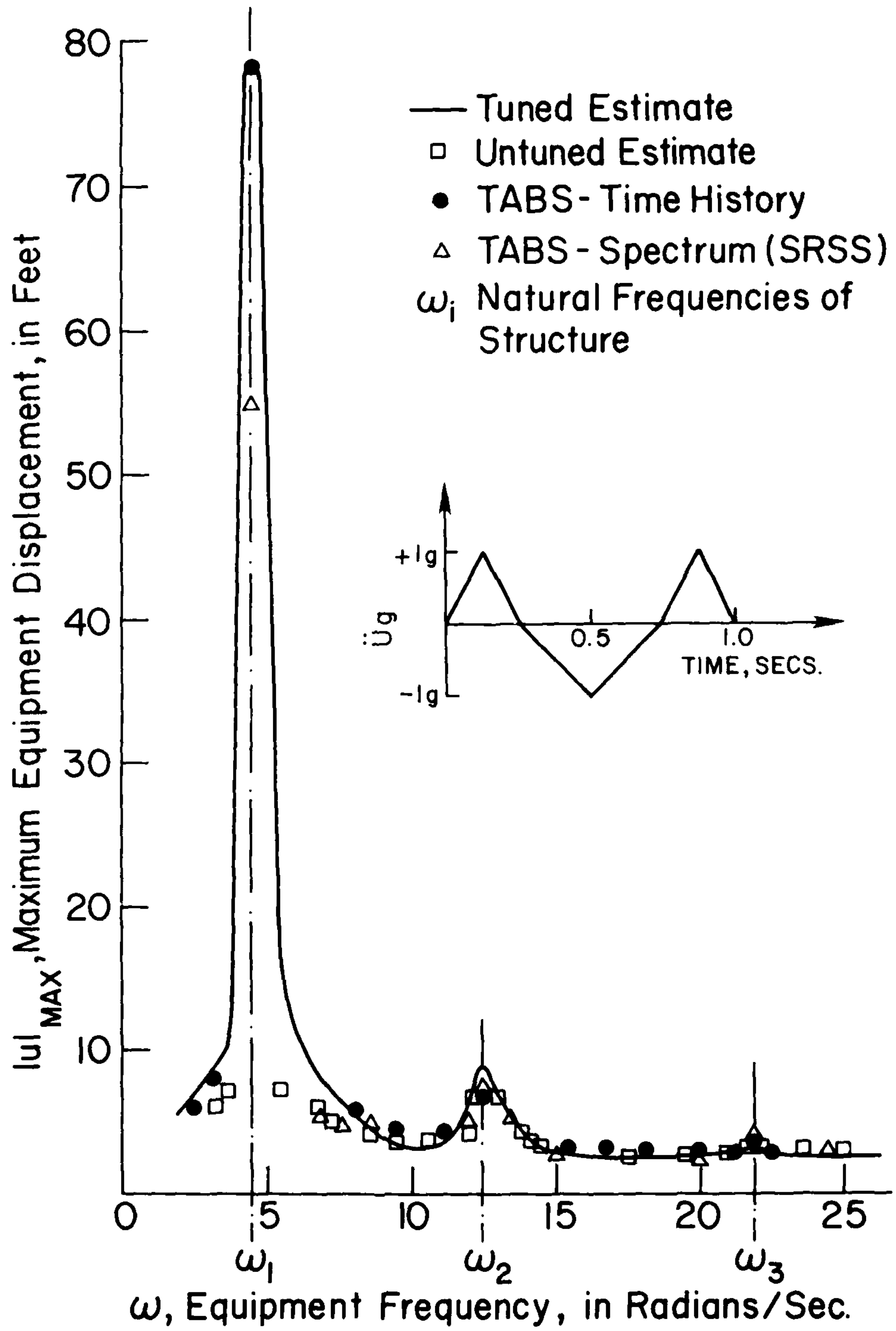
Diagram of Oscillators



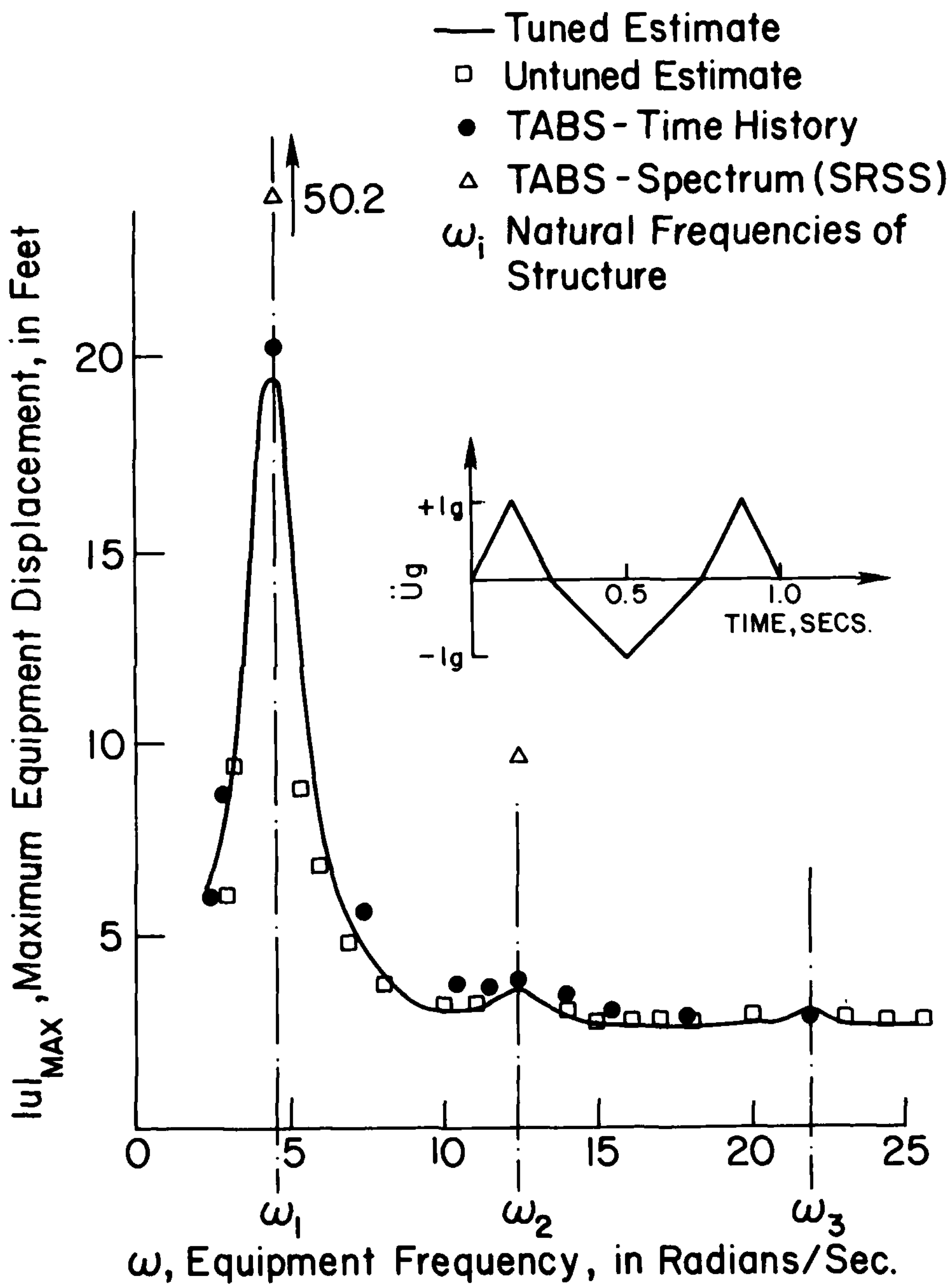
Time History of Acceleration—Shaking Table, Third Floor, and Oscillators



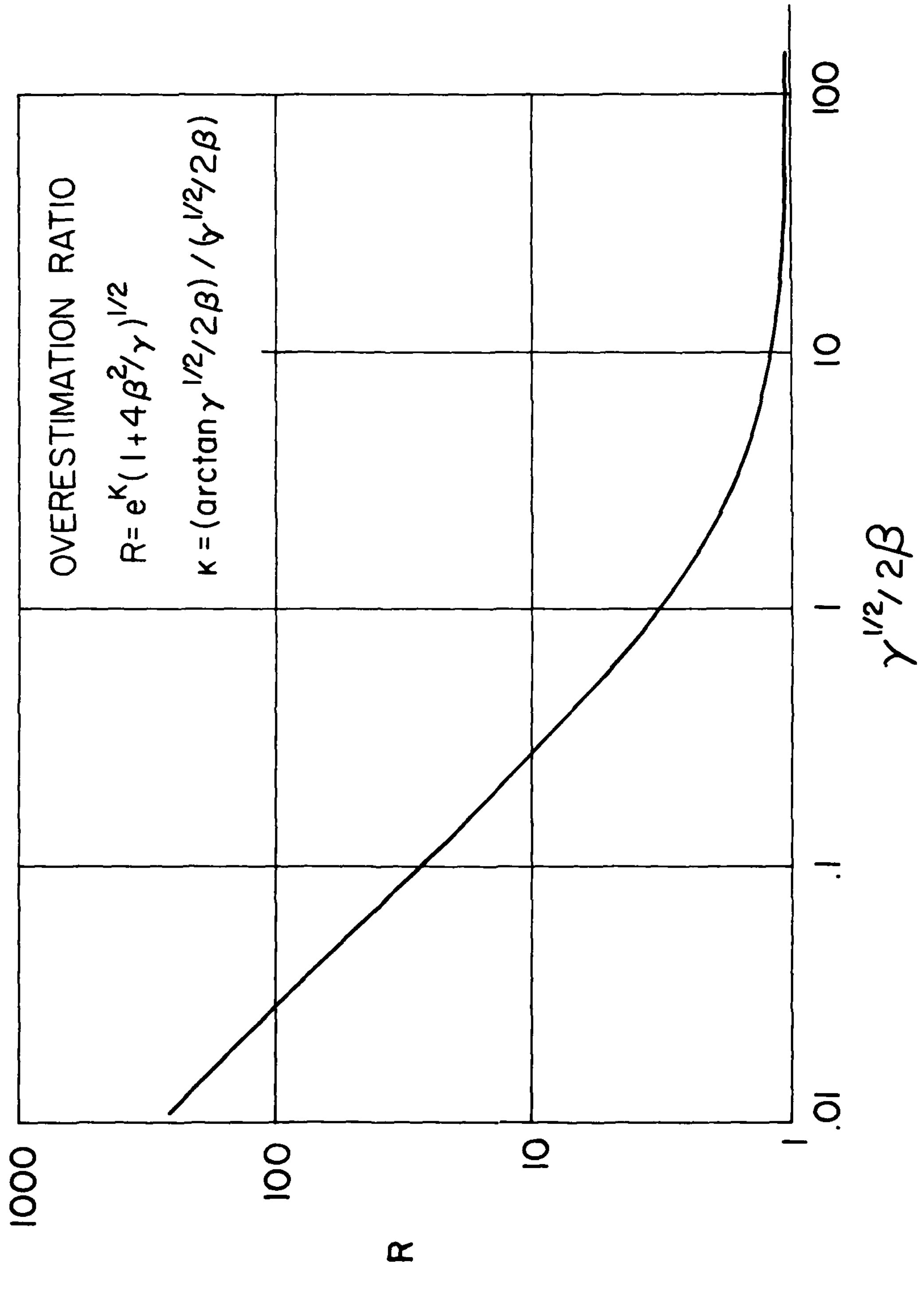
Fourier Transforms of Oscillator Time-History Accelerations



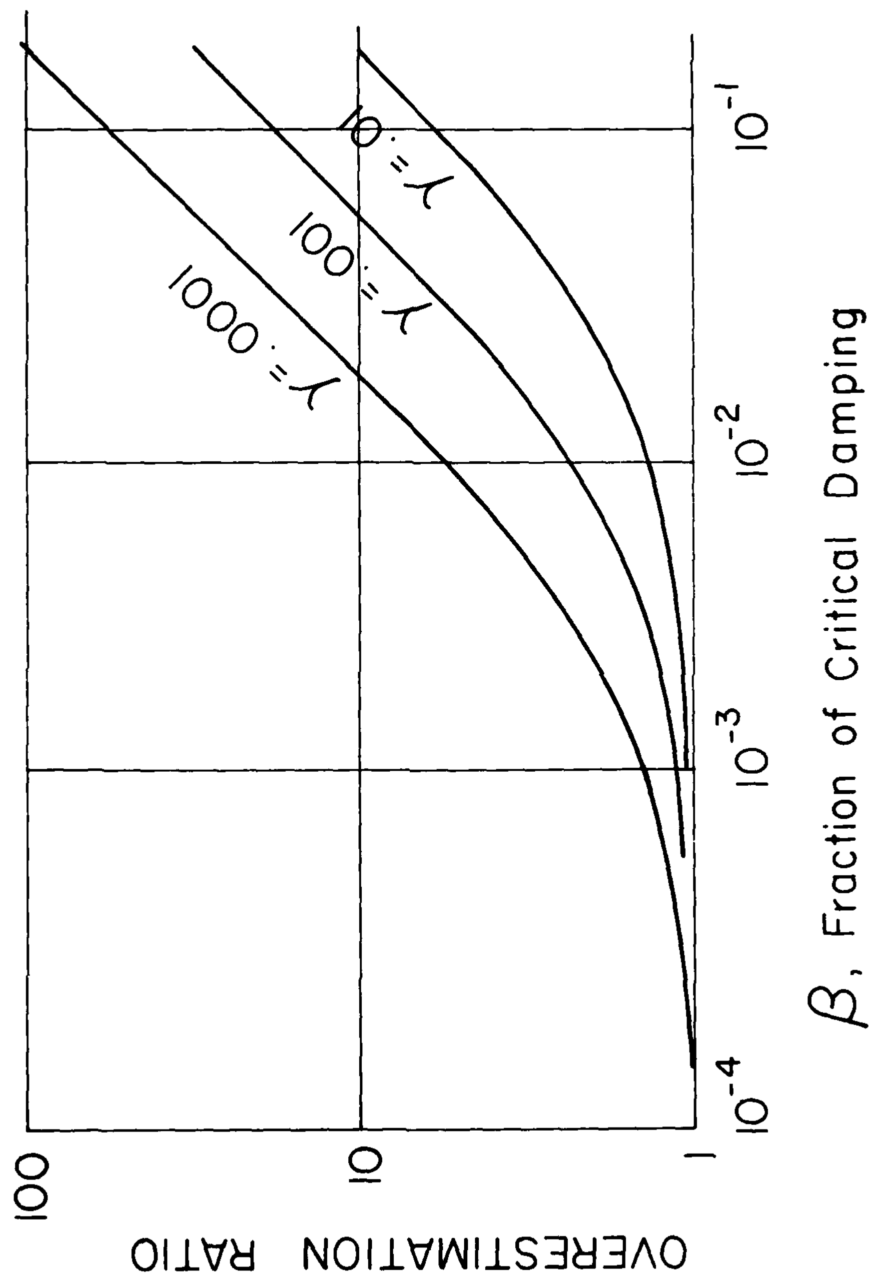
Appendage Response—Short Duration Shock—0% Damping



Appendage Respone—Short Duration Shock—2% Damping



Overestimation Ratio



Overestimation Ratio as a Function of Damping Factor

REFERENCES

[1] J. M. Kelly and J. L. Sackman, "Response Spectra Design Methods for Tuned Equipment-Structure Systems," *Journal of Sound and Vibration,* Vol. 59, No. 2, pp. 171-179 (1978).

[2] J. M. Kelly and J. L. Sackman, "Shock Spectra Design Methods for Equipment-Structure Systems," to appear in *Bulletin of Shock and Vibration* (1979).

[3] N-C. Tsai, "Spectrum-Compatible Motions for Design Purpose," *Journal of the Engineering Mechanics Division,* Proceedings ASCE, Vol. 98, EM2, pp. 345-356 (1972).

[4] J. M. Biggs and J. M. Roesset, "Seismic Analysis of Equipment Mounted on a Massive Structure," in *Seismic Design for Nuclear Power Plants,* edited by R. J. Hansen, MIT Press, Cambridge, Massachusetts (1970).

[5] K. K. Kapur and L. C. Shao, "Generation of Seismic Floor Response Spectra for Equipment Design," *Proceedings,* Structural Design of Nuclear Power Plant Facilities, Chicago, Dec. 17-18, 1973, American Society of Civil Engineers, New York, N. Y. (1973).

[6] M. Amin, *et al.*, "Earthquake Response of Multiply Connected Light Secondary Systems by Spectrum Methods," *Symposium on Seismic Analysis of Pressure Vessel and Piping Components,* ASME, New York, pp. 103-129 (1971).

[7] N. M. Newmark, "Earthquake Response Analysis of Reactor Structures," *Nuclear Engineering and Design,* Vol. 20, No. 2, pp. 303-322 (1972).

[8] A. H. Hadjian, "Some Problems with the Calculation of Seismic Forces on Equipment," *Proceedings,* Structural Design of Nuclear Plant Facilities, Chicago, Dec. 17-18, 1973, American Society of Civil Engineers, New York, N. Y. (1973).

[9] J. Penzien and A. K. Chopra, "Earthquake Response of an Appendage in Multi-Story Building," *Proceedings,* Third World Conference on Earthquake Engineering, Vol. II, New Zealand (1965).

[10] K. A. Peters, D. Schmitz, and U. Wagner, "Determination of Floor Response Spectra on the Basis of the Response Spectrum Method," *Nuclear Engineering and Design,* Vol. 44, pp. 255-262 (1977).

[11] J. L. Sackman and J. M. Kelly, "Rational Design Methods for Light Equipment in Structures Subjected to Ground Motion," *Report No. UCB/EERC-78/19,* Earthquake Engineering Research Center, University of California, Berkeley (1978).

[12] E. L. Wilson and H. Dovey, "Three-Dimensional Analysis of Building Systems," *Report No. EERC 72-8,* Earthquake Engineering Research Center, University of California, Berkeley (1972).

[13] S. W. Zagajeski and V. V. Bertero, "Computer-Aided Optimum Design of Ductile Reinforced Concrete Moment-Resisting Frames," *Report No. UCB/EERC-77/16,* Earthquake Engineering Research Center, University of California, Berkeley (1977).

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